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Some Results on Fuzzy Supra Topological Spaces

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ABSTRACT
In this paper we have obtained some results on fuzzy supra topological spaces introduced in [9].
Keywords- fuzzy supra topological spaces, graph, continuity

I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[10] in 1965 to describe those phenomena which are imprecise or fuzzy in nature. Fuzzy set is a generalization of ordinary set but there is a very significant difference between the two. Due to these differences several set theoretic results are not true in fuzzy setting.

In 1968, Chang[2] introduced fuzzy topology as a natural generalization of ordinary topology. Later on in 1976, R Lowen[] redefined fuzzy topology on a set X in a different way. According to him, a collection τ of fuzzy in X is a fuzzy topology on X if it is closed under arbitrary union, finite intersection and contains all constant fuzzy sets.

In [6], Mashhour et al. introduced the concepts of supra topological spaces, supra open sets and supra closed sets. Later on ME Abd El-Monsef et al [1] introduced the concept of fuzzy supra topological as a natural generalization of the notion of supra topological spaces. Here in this paper we follow the definition given Srivastava and Sinha[9]. They called a family $\tau \sqsubseteq I^x$ a fuzzy supra topology on X if it is closed under arbitrary union and contains all constant fuzzy sets in X.

II. PRELIMINARIES

Here we shall follow Lowen's definition of fuzzy topology[5].I denotes the unit interval [0,1],a constant fuzzy set taking value $\alpha \epsilon [0,1]$ will be denoted by α.A^c will denote the complement of a fuzzy set A in X.α_A

will denote the fuzzy set in X which takes the constant value α on A and zero otherwise.As in [5],a fuzzy point x_r in X is a fuzzy set in X taking value $r\epsilon(0,1)$ at x and zero elsewhere. x and r are called support and value of the fuzzy point x_r respectively. x_r is said to belong to a fuzzy set A in X iff $r<A(x)$.

The following definitions are from [7].A fuzzy singleton x_r in X is a fuzzy set in X taking value $r\epsilon(0,1]$ at x and zero elsewhere. A fuzzy singleton x_r is said to be quasi coincident with a fuzzy set A (notation: $x_r qA$) iff $r+A(x)>1$

Fuzzy supra topology on X is defined earlier by S.Dang et al.[3] and A. Kandil et al.[4] as a subfamily $\tau \sqsubseteq I^x$ which is closed under arbitrary union and contains X,Φ.In[9] Srivastava & Sinha modify this definition as:

Definition 2.1[9] : A subfamily $\tau \sqsubseteq I^X$ is called a fuzzy supra topology on X if it contains all constant fuzzy sets and is closed under arbitrary union.

If τ is a fuzzy supra topology on X,then (X,τ) is called a fuzzy supra topological space, in short , an fsts.

Members of τ are called fuzzy supra open sets (in short,fuzzy S-open sets) and their complements are called fuzzy S-closed sets (in short, fuzzy S-closed sets) in X.

Definition2.2[3]:A fuzzy set A in an fsts is called a fuzzy supra neighbourhood of a fuzzy singleton x_r if $B\epsilon\tau$ such that $x_r \sqsubseteq B \sqsubseteq A$

Definition2.3[3]: Let (X,τ) be an fsts.A subfamily of **B** of τ called a base for τ if each U $\epsilon\tau$ can be expressed as union of members of **B**.

Definition2.4[3]:A mapping $f:(X,\tau_1)\rightarrow(Y,\tau_2)$between two fsts is called fuzzy supra continous (fuzzy S-continuous,in short) if $f^{-1}(v)\epsilon\tau_1$ for every $v\epsilon\tau_2$.

Proposition2.1:A fuzzy set in an fsts (X,τ) is fuzzy S-open iff it is a fuzzy supra neighbourhood of each of its fuzzy points.

Proposition2.2: A fuzzy point $x_r \epsilon UA_i$ iff $x_r \epsilon A_i$ for some i.

Proposition 2.3[9]:A fuzzy set U in an fsts (X,τ) is fuzzy S-open set B such that $x_r \epsilon B \sqsubseteq U$.

Definition2.5[1]: Let (X,τ_1) and (Y,τ_2) be two fsts.Then the product fsts of (X,τ_1) and (Y,τ_2) is defined as the fsts (XxY $,\tau_1 x\tau_2$) where $\tau_1 x \tau_2$ is the fuzzy supra topology on XxY having $\{U_1 x U_2 :U_i ,i=1,2\}$ as a base.

We extend the definition of product fsts in case of an arbitrary family of fuzzy supra topological spaces as follows :

Let $\{ (X_i ,\tau_i):i\epsilon\mathcal{A}\}$ be an arbitrary family of fsts .Then the product fsts $(\Pi X_i,\Pi\tau_i)$ is the one having $\{\Pi U'_I:U'_i=X_i$ except for finitely many i $s\epsilon\mathcal{A}\}$ as a base.

III. FUZZY SUPRA TOPOLOGY AND RELATED CONCEPTS:

Definition3.1:Let (X,τ_1) and (X,τ_2) be two fsts. Then a function f : $(X,\tau_1)\rightarrow(X,\tau_2)$ is said to have a fuzzy S-closed graph if the graph $\mathcal{G}(f) = \{(x,f(x)): x\epsilon X\}$is fuzzy S − closed in XxY.

Proposition3.1: A subset A of the product fsts $(XxY,\tau_1 x\tau_2)$ is fuzzy S-closed iff for each fuzzy point $(x,y)_r\epsilon XxY$-A there exist two fuzzy supra open neighbourhood U and V of x_r and y_r respectively such that (UxV)∩A=Φ

Proof:Let A be fuzzy S-closed in XxY. Then XxY-A is fuzzy S-open.Hence for any fuzzy point $(x,y)_r$ in XxY-A,in view of Proposition 2.3,∃ *a basic* fuzzy S-open set say UxV in XxY such that $(x,y)_r\epsilon UxV\sqsubseteq XxY$-A implying that $x_r\epsilon U$, $y_r\epsilon V$ and (UxV)∩A=φ.

Conversely,let for each fuzzy point $(x,y)_r\epsilon XxY$-A,∃ fuzzy S − open neighbourhood U of x_r and V of $y_r$such that UxV∩A=φ. This implies that for every fuzzy point $(x,y)_r\epsilon XxY$-A,there exists a basic fuzzy S –open set UxV such that $(x,y)_r\epsilon UxV\sqsubseteq XxY$-A and hence XxY-A is fuzzy S-open i.e.A is fuzzy S-closed.

Definition3.2[3]: Let (X,τ) be an fsts and αϵ[0,1). A collection $\mathcal{U} \sqsubseteq I$x is said to be an α-shading of X if for each xϵX,∃ a Gϵu such that G(x)>α. A subcollection of u is called a fuzzy α-subshading if it itself forms a fuzzy α-shading of X.

An α-shading $\mathcal{U}$ of X is called a fuzzy supra open α-shading of X if each member of u is fuzzy supra open.

Definition3.3[3]: An fsts (X,τ) is said to be α-supracompact if every fuzzy supra open α-shading of X has a finite α-subshading.

Definition3.4[3]: Let(X,τ) be an fsts and Y⊑X. Then τ_Y={Y∩A:Aϵτ} is called the fuzzy supra subspace topology on Y and (Y,τ_Y) is called a fuzzy supra subspace of (X,τ).

Definition3.5[9]: An fsts (X,τ) is called Haudorff if for each pair of distinct fuzzy points x_r and y_s in X there exists U,Vϵτ such that $x_r\epsilon U$, $y_s\epsilon V$ and U∩V=φ.

Lemma3.1:Let (X,τ) and (Y,τ^*) be two fuzzy supra topological spaces.Then a function f : (X,τ) → (Y,τ^*) has an fuzzy S-closed graph iff for each pair of fuzzy points x_r ϵX , $y_r\epsilon Y$ such that y≠f(x) there exist two fuzzy supra open sets U and V containing x_r and y_r respectively such that f(U)∩V=φ.

Proof:First let us suppose that f has an S-closed graph i. e.G(f)={(x,f(x)):xϵX} is S-closed in XxY.Then for each $x_r\epsilon X$ and $y_r\epsilon Y$ such that $y\neq f(x)$∃ two fuzzy supra open sets U and V containing x_r and y_r respectively such that (UxV)∩G(f)=φ.

So UxV(x,f(x))=0 ∀ xϵX

Or inf { U(x),V(f(x))}=0 ∀xϵX

We now show that f(U)∩V=φ i.e. for yϵY

(f (U)∩V)(y) =inf { f(U)(y),V(y)}
=inf { $\sup_{x\epsilon f^{-1}(y)}$U(x),V(y)}
=0

……..(2)

This can be seen in the following lines :

(a) If y=f(x) for some x, then from (1),
Inf {U(x) , V(f(x))}= 0

Which implies that either U(x) or V(f(x))=0. Now if V(f(x))=0 then obviously inf $\{\sup_{x\epsilon f^{-1}(y)}$U(x),V(f(x))}=0 and∃if for any xϵX, V(f(x))≠ 0 and from (1),U(x)=0

This is true for any $x\epsilon f^{-1}\{y\}$ (since for any $x\epsilon f^{-1}(y)$,y=f(x) and V(f(x))≠0 so in view of (1) U(x) must be zero)

Therefore $\sup_{x\epsilon f^{-1}(y)}$U(x)=0

and hence f(U)(f(x))= $\sup_{x\epsilon f^{-1}(y)}$U(x)=0

Thus inf $\{\sup_{x\epsilon f^{-1}(y)}$U(x),V(f(x))}=0

Therefore (f(U)∩V)(f(x))=0

Further

(b)for any unmapped element y of Y, f(U)(y)=0 (definition of f(U)). Therefore inf{f(U)(y),V(y)}=0

Thus for all yϵY , inf{f(U)(y),V(y)}=0

Equivalently (f(U)∩V)(y)=0 ∀ yϵY

Or f(U)∩V =φ

Conversely, let $(x,y)_r\epsilon XxY$-G(f). Then y≠f(x) and hence ∃

Fuzzy supra open sets U,V in X such that $x_r\epsilon U$, $y_r\epsilon V$ and f(U)∩V=φ.Now consider UxV.We have

UxV(x,f(x))= inf { U(x),V(f(x))}
≤ inf{supU(x),V(f(x))}
=(f(U)∩V)(f(x))
=0

Thus we have,

$(x,y)_r\epsilon UxV\sqsubseteq XxY$-G(f)

Which shows that XxY-G(f) is fuzzy supra open and hence G(f) is fuzzy supra closed in XxY.

Lemma3.2:Let f be a function from an fsts (X,τ_1) to another fsts (X,τ_2).Then the following statements are equivalent:

(1) f is S-continous.

(2) for each $x_r\epsilon X$ and each τ_2-fuzzy supra open set V⊑Y containing $(f(x))_r$ there exists a τ_1-fuzzy supra open set U⊑X containing x_r such that f(U)⊑V.

Proof:(1)⇒(2)

Since f is S-continuous, the inverse image of each τ_2-fuzzy supra open set is τ_1-fuzzy supra open.Thus for any $x_r\epsilon X$ and any fuzzy open set V of Y containing $(f(x))_r$ there exists $U=f^{-1}(V)\epsilon\tau_1$ such that $f(U)=f(f^{-1}(V))\sqsubseteq V$.

(2) ⇒(1)

Let V be a τ_2-fuzzy supra open set.Then we have to show that $f^{-1}(V)$ is τ_1-fuzzy supra open.Let $x_r\epsilon f^{-1}(V)$.

Then $r<(f^{-1}(V))(x)$
=V(f(x))

So $(f(x))_r\epsilon V$

Therefore, using (2) ∃ $U\epsilon\tau_1$ such that $x_r \epsilon U$ and $f(U)\sqsubseteq V$. Hence $x_r \epsilon U \sqsubseteq f^{-1}(f(U)) \sqsubseteq f^{-1}(V)$ which shows that $f^{-1}(V)$ is a fuzzy supra neighbourhood of each of its fuzzy points. Therefore $f^{-1}(V)\epsilon\tau_1$ (in view of proposition)

Now with the help of above two lemmas we prove the following theorems:

Theorem3.1:If f $:(X,\tau_1)\rightarrow(Y,\tau_2)$ is S-continuous and Y is ST_2-space then f has an S-closed graph.

Proof:Let $(x,y)_r \epsilon XxY-G(f)$. Then $y\neq f(x)$.Now choose $r\epsilon(0,1)$.Since Y is ST_2 , ∃ two fuzzy supra open sets U and V such that $(f(x))_r \epsilon U$, $y_r \epsilon V$ and $U\cap V=\varphi$.Since f is S-continuous,using lemma 3.2 ∃ a fuzzy supra open ∉neighbourhood W of x_r such that $f(W)\sqsubseteq U$. Hence $f(W)\cap V=\varphi$.This implies in view of lemma 3.1 that f has an fuzzy S-closed graph.

Theorem3.2:If $f:(X,\tau_1)\rightarrow(Y,\tau_2)$ is S-continuous injection with an S-closed graph,then X is ST_2.

Proof: Let x_1 ,$x_2 \epsilon X$, $x_1\neq x_2$ then $f(x_1)\neq f(x_2)\Rightarrow(x_1,f(x_2))_r \epsilon XxY-G(f)$.Since f has an S-closed graph, ∃ two fuzzy supra open neighbourhoods U and V of $(x_1)_r$ and $(f(x_2))_r$ respectively such that $f(U)\cap V=\varphi$.Since f is S-continuous,there exists a fuzzy supra open set W containing $(x_2)_r$ such that $f(W)\sqsubseteq V$.Hence $f(W)\cap f(U)=\varphi$ which implies that $W\cap U=\varphi$ and so X is an ST_2-space.

Theorem3.3:Let (X,τ) be a Hausdorff fsts and A be an α-supra compact subset of X.Then any fuzzy point x_r ($x\notin A$) and α_A can be separated by disjoint fuzzy supra open sets of X.

Proof: Let x_r be a fuzzy point and A be a disjoint α-supra compact subset of X.Take $y_\alpha \epsilon A$, then x_r and y_α are two distinct fuzzy points in X and hence ∃ two fuzzy supra open sets say $U_{y\alpha}$ and $V_{y\alpha}$ in X such that $x_r \epsilon U_{y\alpha}$ and $y_\alpha \epsilon V_{y\alpha}$ and $U_{y\alpha}\cap V_{y\alpha}=\Phi$.Now consider $\mathcal{U}=\{A\cap V_{y\alpha}:y_\alpha \epsilon A\}$ Then $\mathcal{U}$ forms an open α-shading of A,therefore ,since A is α-compact∃ a finite α-subshading of A say $\{A\cap V_{y\alpha1},A\cap V_{y\alpha2},\ldots\ldots\ldots A\cap V_{y\alpha n}\}$.

Now consider $\nu=\{V_{y\alpha1,}V_{y\alpha2,}\ldots\ldots\ldots\ldots V_{y\alpha n}\}$.

Take $U=\bigcap_{i=1}^{n} U\alpha i$ and $V=\bigcap_{i=1}^{n} V_{\alpha i}$

Then $x_r \epsilon U$, $\alpha_A \subseteq V$ and further $U\cap V=\Phi$,

Since $U_{\alpha i} \cap V_{\alpha i} =\varphi$

$U_{\alpha n} \cap V_{\alpha n} =\varphi$

Therefore, $(\cap U_{\alpha i}) \cap V_{\alpha i}=\varphi$ for i= 1,2,……….n which implies $(\bigcap_i U_{\alpha i}) \cap (\bigcup_i V_{\alpha i})=\varphi$

Hence, the theorem is proved.

IV. CONCLUSION

Here we have obtained some results in fuzzy supra topological spaces especially related to graph and continuity.

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